

Dark Energy as the Remnant of Inflation

A Physical Basis for Inflation and Dark Energy

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Abstract

A QED-based symmetry breaking/bootstrap mechanism, appearing at sufficiently small space-time distances, is suggested as an explanation for the vacuum energy that furnished the initial impulse for Inflation, and continues on, to the present day, to provide the “ Dark Energy ” which is apparently forcing our Universe apart. Very high frequency virtual vacuum currents are assumed to generate weak, effective electromagnetic fields, corresponding to the appearance of an effective 4-potential $A_\mu^{\text{vac}}(x)$, which is itself equal to the vacuum expectation value of the operator $A_\mu(x)$ in the presence of that $A_\mu^{\text{vac}}(x)$. Lorentz invariance is manifest, as every observer would measure the same electric field in his or her own reference frame. Such an effective vacuum field would have no relevance to the motion of ordinary charged particles until particle energies on the order of 10^5 TeV are possible. The model is sufficiently constrained so that one parameter is needed to fit the vacuum energy densities and relevant times for the onset and end of Inflation, as well as those parameters of present day Dark Energy.

1 Introduction

Every so often, one is struck by an idea which is so simple and compelling that it is hard to understand why it has been overlooked. In our considerations of the very small, we have come across such an idea, one which has astrophysical implications on a grand scale, even surpassing the two topics mentioned in the title.

The essence of this very simple idea is as follows. Conventionally, electromagnetic fields are either “quantized” or “external”, and by external is meant classical fields which can be switched on and off. Simultaneously, one speaks of the “quantum mechanical vacuum”, in which quantized fields of arbitrary complexity are fluctuating, and whose effects can only be indirectly inferred (such as the 27 Megacycles of the 1057 Megacycle Lamb shift separating the $2S^{1/2}$ and $2P^{1/2}$ levels of the Hydrogen atom).

Imagine that one had available a “super Heisenberg” gamma-ray microscope, so that one could “see” a virtual photon of the QM vacuum suddenly transform itself into a bubble corresponding to the virtual appearance of an electron and positron, whose propagations define the sides of the bubble, and which propagation continues for a duration on the order of the inverse of their mass. When these virtual particles are separated there is an electric field between them, which can be thought of an electromagnetic fluctuation that disappears when the bubble collapses. Such fluctuations are present in the QM vacuum; they cannot be “turned off” and, in an appropriately averaged way, they should contain energy. They arise from the fundamental quantization of the operator QED fields. The averaged or background electromagnetic energy of these fluctuations might well be characterized by the existence of an effective, c-number A_μ^{vac} which is always present. The existence of such an averaged, effective field, due to its QM origin, could possibly have striking effects on the classical world about us.

The implications of this idea encompass a theory of dark matter, gamma ray bursts (including an interpretation of the recent one [1] which blinded the Swift observations), ultra-high energy cosmic rays (which are able to violate the GZK limit), and even wide angle CMB fluctuations. In this paper, we shall explain, in some detail, the initial mechanism which easily accounts for a simple understanding of both inflation and dark energy, corresponding to a new, symmetry breaking, bootstrap solution in QED; and follow this by reference to an existing hep-th arXiv submission [2], which describes the remaining material. A previous arXiv submission [3] described a different symmetry breaking solution, but one which is relevant only to dark energy; the present, “improved” solution provides a simple, physical picture for the onset and duration of inflation, as well as present day dark energy.

One prejudice of the authors should be stated at the outset, which forms part of the motivation for the present remarks. A conventional approach to vacuum energy is that the latter represents, in some fashion, zero point energies of relevant quantum fields. Aside from being horribly divergent, and so requiring massive renormalizations, it is not clear that zero point terms even belong in any field theory, for they can be well understood as the remnants of improper positioning of products of operators at the same space-time point. One cannot proceed from classical to quantum forms without facing this question, which has long ago been given a Lorentz invariant answer by Symanzik [4] in terms of “normal ordering”. When Lorentz invariant is subsumed into a more general relativistic invariance, in which

all energies couple to the metric, there is still no requirement of including those zero point terms which should have been excluded at the very beginning; instead, zero point terms have been pressed into service as the simplest way of attempting to understand vacuum energy.

Because zero point energies have been used to give a qualitative explanation of the logarithm of the lowest order Lamb shift does not mean that they are the correct explanation for that effect; zero point energies cannot produce the additive constant to that logarithm, and they most certainly cannot produce the higher order terms which have been experimentally verified. It is here suggested that there is another source of vacuum energy, one which violates no principles of quantum field theory, that can provide a simple and at least qualitatively reasonable QED mechanism for both inflation, occurring at the beginning of our Universe, and for its remnant dark energy, which continues that acceleration at a greatly reduced and gradually declining rate.

2 Formulation

We take the point of view that such an averaged $A_\mu^{\text{vac}}(x)$ can be treated as an effective, classical, external field, albeit one which is always present, and should be included in QED considerations, and specially in the construction of the QED Generating Functional (GF). There, the Schwinger–Symanzik–Fradkin formulations [4] produce a GF, $\mathcal{Z}[j, \eta, \bar{\eta}]$, which, for our purposes, can be transformed into the most convenient form :

$$\begin{aligned} \mathcal{Z}[\eta, \bar{\eta}, j] &= \mathcal{N} \exp \left[\frac{i}{2} \int d^4x d^4y j^\mu(x) D_{c,\mu\nu}(x-y) j^\nu(y) \right] \\ &\times e^{\mathcal{D}_A} \exp \left[i \int d^4x d^4y \bar{\eta}(x) G_c(x, y|A) \eta(y) \right] \exp[L[A]] \end{aligned} \quad (2.1)$$

with $A_\mu(x) = \int d^4y D_{c,\mu\nu}(x-y) j^\nu(y)$.

The relativistic notation used throughout this article is the so-called “east coast metric”, with the scalar product defined by $ab = \vec{a}\vec{b} - a_0b_0$, and the Dirac matrices such that $\gamma_\mu^\dagger = \gamma_\mu$, $\gamma_\mu^2 = 1$ and $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$.

The normalization $\mathcal{Z}[0, 0, 0] = 1$ is defined by : $\mathcal{N}^{-1} = \langle 0|S|0 \rangle = e^{\mathcal{D}_A} \exp[L[A]] \Big|_{A=0}$, $G_c[A] = [m + \gamma \cdot (\partial - ieA)]^{-1}$ is the Feynman (causal) electron propagator in an arbitrary external field $A_\mu(x)$; $L[A] = \text{Tr} \ln(G_c^{-1}[A] G_c[0])$ is the vacuum functional corresponding to a single closed lepton loop, to which is attached all possible (even) number of fields A_μ ; and $D_c(x-y)$ is the free photon propagator in an arbitrary relativistic gauge. We shall refer to the quantity :

$$e^{\mathcal{D}_A} \quad \text{with} \quad \mathcal{D}_A = -\frac{i}{2} \int d^4x d^4y \frac{\delta}{\delta A_\mu(x)} D_{c,\mu\nu}(x-y) \frac{\delta}{\delta A_\nu(y)}$$

as the linkage operator, for its function is to link all pairs of A -dependence upon which it acts by virtual D_c propagators. We begin this discussion specifying those radiative corrections due to a virtual electron–positron bubble, but will later generalize to the corresponding situations when similar contributions are made by muon and tau bubbles.

Eq. (2.1) as written is true when no classical, external field is present; however if such fields were present, (2.1) would still be true if the A_μ in G_c and L were replaced by $A_\mu + A_\mu^{\text{ext}}$. In this way, it is easy to see the relation between QED and Quantum Mechanics : were the A dependence and the linkage operator suppressed, one has the GF for many-body Potential Theory, where the fermions are moving in the potential A_μ^{ext} . All of the virtual structure of QED corresponds to the action of that linkage operator, connecting all the A dependence upon which it acts, in all possible ways.

Let us now assume the existence of an effective, classical field arising from the virtual fluctuations of the QED vacuum; for the present discussion, no ordinary, classical, external field need be included. Conventionally, the vacuum expectation value (vev) of the current operator $j_\mu(x) = ie\bar{\psi}(x)\gamma_\mu\psi(x)$ must vanish in the absence of classical, external fields, designated by $A_\mu^{\text{ext}}(x)$:

$$\langle 0 | j_\mu(x) | 0 \rangle \Big|_{A^{\text{ext}}=0} = 0$$

When such a classical A_μ^{ext} is present, the current it induces in the vacuum can be non zero :

$$\langle 0 | j_\mu(x) | 0 \rangle \Big|_{A^{\text{ext}} \neq 0} \neq 0$$

although strict current conservation demands that $\partial_\mu \langle 0 | j_\mu(x) | 0 \rangle \Big|_{A^{\text{ext}} \neq 0} = 0$.

The conventional mathematical apparatus used to describe the vacuum state, or vev of currents operators, makes no reference to the scales on which vacuum properties are to be observed; and in this sense, it is here suggested that the conventional description is incomplete. On distance scale larger than the electron's Compton wave length, $\lambda_e \sim 10^{-10}$ cm, one can imagine that the average separation of virtual e^+ and e^- currents is not distinguishable, and hence that the vacuum displays not only a zero net charge, but also a zero charge density. But on much smaller scales, such as 10^{-20} cm, the average separation distances between virtual e^+ and e^- are relatively large, with such currents describable as moving independently of each other – until they annihilate. Since there is nothing virtual, or “ off shell ”, about charge, on sufficiently small space-time scales such “ separated ” currents can be imagined to produce “ effective c-number ” fields, characterized by an $A_\mu^{\text{vac}}(x)$, which could not be expected to be measured at distances larger than λ_e , but which exist and contain electromagnetic energy on scales much smaller than λ_e . We therefore postulate that, at such small scales and in the absence of conventional, large scale A_μ^{ext} , $\langle 0 | j_\mu(x) | 0 \rangle$ need be neither x -independent nor zero; but rather, that it generates an $A_\mu^{\text{vac}}(x)$ discernible at such small scales, which is given by the conventional expression :

$$A_\mu^{\text{vac}}(x) = \int d^4y D_{c,\mu\nu}(x-y) \langle 0 | j_\nu(y) | 0 \rangle \quad (2.2)$$

where $D_{c,\mu\nu}$ is the usual, free field, Feynman (causal) photon propagator that, for convenience, is defined in the Lorentz gauge, $0 = \partial_\mu A_\mu^{\text{vac}} = \partial_\mu D_{c,\mu\nu} = \partial_\nu D_{c,\mu\nu}$.

For comparison, note that classical electromagnetic vector potentials can always be written in terms of well defined, classical currents J_μ , by an analogous relation : $A_\mu = \int D_{c,\mu\nu}(x-y) J_\nu$, while the transition to operator QED in the absence of conventional, large scale, external fields involves the replacements of the classical vector potential and

currents by operators $A_\mu(x)$ and $j_\mu(x) = ie\bar{\psi}(x)\gamma_\mu\psi(x)$, which satisfy operator equations of motion : $(-\partial^2)A_\mu(x) = j_\mu(x)$, or :

$$A_\mu = \int D_{c,\mu\nu}(x-y) j_\nu + \hat{A}_\mu \quad (2.3)$$

where \hat{A} denotes a free field operator satisfying $(-\partial^2)\hat{A} = 0$; its vev is zero.

Hence, calculating the vev of (2.3) yields :

$$\langle 0 | A_\mu(x) | 0 \rangle = \int d^4y D_{c,\mu\nu}(x-y) \langle 0 | j_\nu(y) | 0 \rangle \quad (2.4)$$

and conventionally, in the absence of the usual, large scale external fields, both sides of (2.4) are to vanish. However, if we assume that non zero $\langle 0 | j_\mu(x) | 0 \rangle$ can exist on ultra short scales, then a comparison of (2.4) with (2.2) suggests that the $A_\mu^{\text{vac}}(x)$ produced by such small scale currents are to be identified with $\langle 0 | A_\mu(x) | 0 \rangle$ found in conventional QED in the presence of the same $A_\mu^{\text{vac}}(x)$. In other words :

$$\begin{aligned} A_\mu^{\text{vac}}(x) &= \langle 0 | A_\mu(x) | 0 \rangle = \frac{1}{i} \int d^4y D_{c,\mu\nu}(x-y) \frac{\delta}{\delta A_\nu(y)} e^{-\frac{i}{2} \int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A}} \\ &\times \langle 0 | S[A^{\text{vac}}] | 0 \rangle^{-1} \exp[L[A + A^{\text{vac}}]] \end{aligned} \quad (2.5)$$

which provides a bootstrap equation with which to determine such short scale $A_\mu^{\text{vac}}(x)$, if any exist. In (2.5), which can be transform into a functional integral relation, the vacuum to vacuum amplitude is given by [4] :

$$\langle 0 | S[A^{\text{vac}}] | 0 \rangle = e^{-\frac{i}{2} \int \frac{\delta}{\delta A} D_c \frac{\delta}{\delta A} \exp[L[A + A^{\text{vac}}]]} \Big|_{A \rightarrow 0} \quad (2.6)$$

Of course, one immediate solution to (2.5) is $A_\mu^{\text{vac}}(x) = 0 = \langle 0 | A_\mu(x) | 0 \rangle$, the conventional solution. But we are interested in, and shall find, solutions that may be safely neglected at conventional nuclear and atomic distances, but which are non zero in an interesting way at much smaller distances. In a sense, such non zero solutions are akin to those found in symmetry breaking processes, such as spontaneous or induced magnetization; qualitatively similar ideas have previously been discussed elsewhere [5] for other reasons.

3 Approximation

How does one go about finding a solution (2.5) ? The first requirement is a representation for $L[A + A^{\text{vac}}]$ which is sufficiently transparent to allow the functional operation of (2.5) to be performed; but, sadly, this is still a distant goal, which has been approximatively realized in only a few, special cases [6]. What shall be done here is to use the simplest, renormalized perturbative approximation to L :

$$L[B] \longrightarrow \frac{i}{2} \int d^4x d^4y B^\mu(x) K_{\mu\nu}(x-y) B^\nu(y)$$

where the $K_{\mu\nu}$ corresponds to using the simplest, first order, renormalized, closed electron loop contribution :

$$\tilde{K}_{\mu\nu}(k) = (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \Pi(k^2) , \quad k^2 = \vec{k}^2 - k_0^2$$

and :

$$\Pi(k^2) = \frac{2\alpha}{\pi} \int_0^1 du u(1-u) \ln[1 + u(1-u) \frac{k^2}{m^2}] \equiv \phi(k^2/m^2) \quad (3.1)$$

where $\alpha = e^2/4\pi \simeq 1/137$ denotes the renormalized coupling, and m is the electron mass. Higher order perturbative terms should each yield a less important contribution to the final answer, although the latter could be qualitatively changed by their sum; we shall assume that this is not the case, and that the perturbative approximation (properly unitarized by the functional calculation which automatically sums over all such loops) gives a qualitatively reasonable approximation.

The functional operation of (2.5), now equivalent to Gaussian functional integration, is immediate and yields the approximate relation for this $A_\mu^{\text{vac}}(x)$:

$$A_\mu^{\text{vac}}(x) = \int d^4y \left(D_c K \frac{1}{1 - D_c K} \right)_{\mu\nu} (x-y) A_\mu^{\text{vac}}(y)$$

or :

$$\tilde{A}_\mu^{\text{vac}}(k) = \left(\Pi(k^2) \frac{1}{1 - \Pi(k^2)} \right) \tilde{A}_\mu^{\text{vac}}(k) \quad (3.2)$$

The simplest non zero solution to (3.2) may be found in the “ tachyonic ” form :

$$\tilde{A}_\mu^{\text{vac}}(k) = C_\mu(k) \delta(\vec{k}^2 - k_0^2 - M^2) = C_\mu \delta(k^2 - M^2) \quad (3.3)$$

which then requires that :

$$1 = \Pi(M^2) \frac{1}{1 - \Pi(M^2)} \quad \Pi(M^2) = \frac{1}{2}$$

or : $\phi(\frac{M^2}{m^2}) = \frac{\pi}{4\alpha}$ which serves to determine M . Note that a solution of form $C_\mu \delta(k^2 + \mu^2)$ would not be possible, since the log of $\Pi(-\mu^2)$ picks up an imaginary contribution for time like $k^2 = -\mu^2$, for $\mu > 2m$.

An elementary evaluation of the integral of (3.1) for large M/m yields :

$$\phi(\frac{M^2}{m^2}) \simeq \ln\left(1 + (\frac{M}{2m})^2\right) + O(1) \simeq 2 \ln(\frac{M}{m}) + \dots \quad (3.4)$$

so that $M = 2m \exp(\frac{\pi}{8\alpha})$, neglecting relatively small corrections. Since $\frac{\pi}{8\alpha} \simeq 54$, (3.4) yields a value for M a bit above the Planck mass, $M \simeq 10^{20} \text{ GeV}/c^2$.

If the muon self energy bubbles are included, the numerical situation changes, and somewhat more favorably, because the value of M found above decreases to 10^{10} GeV ; and if the tau bubbles are also included, M drops to approximately 10^8 GeV . All three

M values correspond to wave numbers much larger than those associated with atomic or nuclear energies. For later convenience, we shall label these three estimates, in descending order of magnitude, as M_0 , M_1 , and M_2 .

The reason that quark–antiquark vacuum polarization bubbles have not been included is that those processes would be dominated by strong coupling gluonic effects, with a well understood mechanism of attraction between quark and antiquark, one that is far stronger than the coulombic one; and hence one might expect strong couplings that could “ overpower ” these electromagnetic effects. This, in turn, suggests that a non perturbative understanding of color charge renormalization in QCD is a prior necessity to the estimation of such electromagnetic radiative corrections; if, for example, a quark binding potential was active, suppressing the size and duration of such quark –antiquark bubbles, their electromagnetic contribution to A_μ^{vac} should be minimal.

4 Computation

In order to fully describe this A_μ^{vac} , one must define $C_\mu(k)$. Remembering that the simplifying choice of a Lorentz gauge has already been made for this vacuum potential, one might expect to be able to choose :

$$C_\mu(k) = v_\mu - k_\mu(k_\nu v_\nu)/k^2$$

where v_μ is a constant to be determined. But the part proportional to k_μ is a pure gauge term, which cannot contribute to any electromagnetic field, and is therefore irrelevant, while the replacement of $C_\mu(k)$ by v_μ serves to generate fields that diverge in the region of the light cone. This is the form of solution used in ref[3], which required an ad hoc cut off when computing energies.

A far better solution is obtained by enforcing the Lorentz gauge condition with the replacement of $C_\mu(k)$ by $\kappa v_\mu \delta(k \cdot v)$ where κ is a constant to be determined; this choice produce a vacuum field that is both simple and everywhere finite. But before such a solution can be taken seriously, there are certain questions that must be answered :

a) What is v_μ ? Physically, that vector should represent an electric field polarization, corresponding to the fluctuating electric field in the plane of the fluctuating bubbles. But the QED vacuum will have bubbles fluctuating in all possible planes, or even on curved surfaces; and there can be no \vec{v} direction singled out. It is then intuitively clear that v_μ should have only a fourth component.

b) But in which Lorentz frame ? If it is to represent a field generated by the same vacuum processes in every frame, it should have the same value to each observer in his own Lorentz frame.

We now show that this choice of solution for the vacuum field does satisfy both of these requirements, and has just the correct behavior to suggest a mechanism for inflation, while producing a present day energy density that can be associated with dark energy.

Insert a representation for both delta functions of :

$$\tilde{A}_\mu^{\text{vac}}(k) = \kappa v_\mu \delta(k \cdot v) \delta(k^2 - M^2) \quad (4.1)$$

and calculate the Fourier transform of $\tilde{A}_\mu^{\text{vac}}$. It will be represented by the parametric, “proper time” integral :

$$A_\mu^{\text{vac}}(x) = \frac{\kappa}{(2\pi)^4} \sqrt{\frac{i\pi}{4}} \left(\frac{-iv_\mu}{\sqrt{v^2}} \right) \int_{-\infty}^{+\infty} ds s^{-3/2} \epsilon(s) e^{isM^2 + iX^2/4s} \quad (4.2)$$

with $\epsilon(s) = \frac{s}{|s|}$, $x \cdot v = \vec{r} \cdot \vec{v} - x_0 v_0$, and where $X^2 = x^2 - (x \cdot v)^2/v^2$, and is a Lorentz invariant quantity. The corresponding solution in another Lorentz frame, represented by a prime on x and a prime on v , will have exactly the same form.

Now, consider the solution (4.2) in our frame, and ask what value should be assigned to the spatial components. From the argument of a) above, the only sensible value for this field A_μ^{vac} is the choice $\vec{v} = 0$. Then, the quantity X^2 reduces to r^2 , and (4.2) can be evaluated trivially :

$$A_\mu^{\text{vac}}(x) \longrightarrow A_4^{\text{vac}}(x) = i \frac{\kappa}{(2\pi)^3} \epsilon(v_0) \frac{\sin(Mr)}{r} \quad (4.3)$$

and depends only on r and on the sign of v_0 .

Now switch to another Lorentz frame, where the result is given by (4.2) using prime variables, related to the unprimed variables by standard Lorentz transformation. An observer in that frame asks what value he should assign to the spatial components of his v' ; and for his vacuum field, he comes to exactly the same conclusion as did we : the only sensible choice for a vacuum field as seen by him must require $\vec{v}' = 0$. Note that his square root variable $\sqrt{X'^2}$ containing all the x' dependence is a Lorentz invariant quantity, and is equal to our square root variable. In our frame, when we set $\vec{v} = 0$, that variable reduces to r ; and in his frame, when he sets $\vec{v}' = 0$ it reduces to r' . But both must be equal, since they were derived from the same invariant; when the observer in the primed frame sets his $\vec{v}' = 0$, as he must to describe his vacuum field, he is using the same functional form as (4.3) in terms of his r' .

The only possible difference between the two expressions of (4.3), primed or unprimed, is the sign of v_0 and that of v'_0 . But, as always when dealing with physical entities, we restrict all admissible Lorentz transformations to those which are orthochronous, keeping the same sense of time, or of energy, or in this case of v_0 ; and hence $\epsilon(v_0) = \epsilon(v'_0)$ and the two versions of (4.3) are the same. In this way, observers in every Lorentz frame see the same vacuum field. For simplicity, we shall choose $\epsilon(v_0) = +1$, although this choice of sign has no bearing on the vacuum energy densities to be calculated.

5 Application to Inflation

Following the arguments above, any observer in any frame will see a “vacuum electrostatic” potential of form :

$$\phi^{\text{vac}}(r) = \kappa' \frac{\sin(Mr)}{r}, \quad \kappa' = \frac{\kappa}{(2\pi)^3}$$

where κ' is a constant to be determined. A simple computation following (3.4) shows that the constant M can have three different, order of magnitude values : for electron bubbles

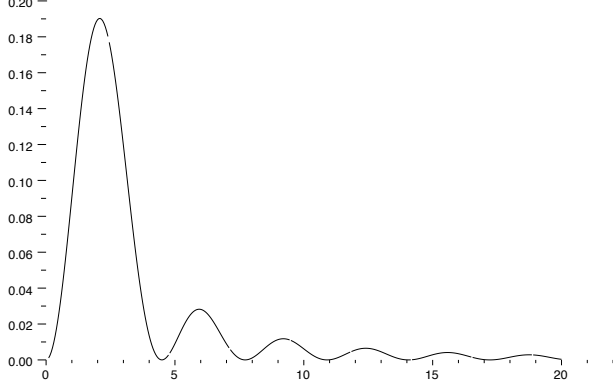


Figure 1: A plot of $f(x) = \frac{1}{x^2} \left(\cos x - \frac{\sin x}{x} \right)^2$ vs. x

only, $M_0 \simeq 10^{20}$ GeV, for electron plus muon bubbles, $M_1 \simeq 10^{10}$ GeV, and for electron, muon and tau bubbles, $M_2 \simeq 10^8$ GeV. The resulting electric field has a rapid spatial variation, as does its energy density :

$$\rho = \vec{\mathcal{E}}^2 / 8\pi = \xi \frac{M^2}{r^2} \left(\cos(Mr) - \frac{\sin(Mr)}{Mr} \right)^2 = \xi M^4 f(x) \quad (5.1)$$

where $\xi = \kappa'^2 / 8\pi$, $x = Mr$, and $f(x) = \frac{1}{x^2} \left(\cos x - \frac{\sin x}{x} \right)^2$. A plot of $f(x)$ has the form indicated in Fig.1, which suggests the basic idea of this approach : the first pulse serves to kick start inflation, which is supposed to begin at $t \sim 10^{-42}$ s, and have an average energy density ρ such that $\rho^{1/4} \sim 10^{18}$ GeV. These numbers, and the time when inflation stops, $t \sim 10^{-32 \pm 6}$ s, with an average $\rho^{1/4} \sim 10^{13 \pm 3}$ GeV, are from Table 2.1 of Liddle and Lyth [7]. However, the initial $\rho^{1/4}$ has simply been specified as the Planck mass, with no uncertainties attached, for the Planck mass just symbolizes the beginning of inflation; in reality, several orders of magnitude of uncertainties should be associated with that number, or with any such number for which a model exists. In the present case, there is such a number, $M_0 \sim 10^{20}$ GeV, two orders of magnitude larger than the Planck mass.

If the charged lepton bubbles are the only ones considered, the question then arises : those of which leptons, and when ? Consider first the present day, almost static situation, of a gradually accelerating Universe whose basic QM holds in a smoothly probabilistic way. Here, one can imagine that the electron bubbles are the most important on the basis of the Uncertainty Principle, since the duration of such quantum fluctuations is roughly proportional to the inverse of the total energy of that fluctuation – here proportional to the lepton masses of each bubble. A tau bubble fluctuation should exist for a far shorter time than a muon bubble fluctuation, and the duration of both should be far less than that of an electron bubble fluctuation. Were a probability parameter to be associated with each type of fluctuation, the total A_μ^{vac} field would be essentially that of the electron.

In contrast, consider the violent, explosive appearance of matter, real and virtual, during that initial pulse of Fig.1; in this model, inflation is associated with the violent injection of

matter into the then tiny Universe during that first pulse. At the beginning of inflation, and of the resulting A_μ^{vac} – at the beginning of that first pulse – the electron bubble fluctuation may have been dominant; but almost immediately there came an explosive torrent of muons and taus such that this first pulse became an almost even mixture of all charged leptons. To describe the short duration of this explosive phase, static probability theory is useless; here the quantum effects of all three lepton bubbles acting simultaneously must be considered.

We make a simple idealization of what happened – one which is adequate for an order of magnitude estimation – by assuming the following crude picture : electron bubbles were responsible for the rise of that first pulse, but during the fall of that pulse, all three bubbles were equally active. This is crude, but allows a simple calculation; and we will find that it is sufficient to describe the orders of magnitude of the initial and final times of inflation, and the corresponding energy densities at those times.

The above idealization is justified by the physical picture which adopts the 10^{20} number at the very instant of creation, immediately following the Big Bang : electron/positron, as the lightest of the leptons would presumably be the first virtual bubbles created, and it is their vacuum energy, that supplies the initial kick to the Universe which we call inflation. By the time inflation ends, virtual muon and tau bubbles should be available, and the order of magnitude M value has dropped to 10^8 GeV. This intuitive picture of the relevant changes in vacuum energy, in the explosion of energy immediately following the Big Bang, defines the present model in so far as it relates to inflation.

Our crude method for the estimation of orders of magnitude assumes that, in the first half of the first pulse of Fig.1, $M \sim 10^{20}$ GeV, which value quickly drops to 10^8 GeV when estimating the second half of that first pulse. That is, inflation is supposed to be in full swing during the growing half of that first pulse, and is heading towards an end at the close of the second half of that pulse. Inspection of Fig.1 shows that the energy in that first half pulse is roughly :

$$U \simeq (.4)(4\pi) \xi M$$

and when divided by the volume $(4\pi/3)R^3 = (4\pi/3)(2/M)^3$ yields for the average energy density :

$$\rho \simeq (.15) \xi M^4$$

and we henceforth choose $\xi \sim 1$, so that :

$$10^{19} < \rho^{1/4} < 10^{20} \text{ GeV} \quad (5.2)$$

The corresponding t value for the onset of inflation is :

$$t = \frac{R}{c} = \frac{x}{c} \frac{1}{M} = \frac{x}{c} \frac{1}{m} \left(\frac{m}{M} \right)$$

where m is the electron mass and R is the radius of the Universe at that time. Passing to cgs units :

$$t = \frac{x}{c} \left(\frac{\hbar}{mc} \right) \left(\frac{m}{M} \right) \sim \frac{x}{3} \cdot 10^{-10} \cdot 10^{-10} \cdot 10^{-23} = \frac{x}{3} \cdot 10^{-43} \text{ s} \quad (5.3)$$

and for $1 < x < 2$, this works nicely.

Inflation is supposed to end at $t \sim 10^{-32 \pm 6}$ s, and in this model that corresponds to the onset (crudely speaking, at half width of the initial pulse) of “ heavier ” muon and tau bubbles, where M has fallen to 10^8 GeV. The total amount of energy in that entire pulse is roughly the same as that of (5.2) –because M_0 is 12 orders of magnitude larger than M_2 – but ρ is smaller since $(4\pi/3)R^3$ is larger. Assuming that all three lepton bubbles are now active, the relation between t and x is changed to read :

$$t \sim \frac{x}{3} \cdot 10^{-10} \cdot 10^{-10} \cdot 10^{-11} = \frac{x}{3} \cdot 10^{-31} \text{ s}$$

and for $2 < x < 4$, this is rather good agreement.

Finally, the simplest ρ estimate at the end of inflation is given by using the previous first pulse energy, but divided by the new and larger volume; using subscripts 0 and 2 for initial and final inflation values, this gives :

$$\rho_2 \simeq (.15) \xi M_0^4 R_0^3 / R_2^3 = (.15) \xi M_0^4 (t_0/t_2)^3 \sim 10^{-37} M_0^4$$

or :

$$\rho_2^{1/4} \sim 10^{10.75} \text{ GeV} \quad (5.4)$$

which overlaps with the expected $\rho_2^{1/4} \sim 10^{13 \pm 3} \text{ GeV}$.

6 Application to Dark Energy

It was assumed above that electrons initially dominated the explosive output of the Big Bang, but that there were so many muons and taus produced in the final mixture of the first pulse, that the effective M value dropped to 10^8 GeV, in the presence of the three leptons. A qualitative, Uncertainty Principle argument was presented in the previous Section which suggests that present day estimates of A_μ^{vac} are due mainly to virtual electron bubbles, for which the M value $\sim 10^{20}$ GeV. If we therefore compute the energy density at present times, either by integrating the energy of the electrostatic vacuum field and dividing by the volume of a sphere of radius R_{now} , or by extracting the non oscillatory behavior of (5.1), one finds :

$$\rho_{\text{now}} \sim M_0^2 / R_{\text{now}}^2 \rightarrow M_0 \left(\frac{M_0 c}{\hbar} \right) R_{\text{now}}^{-2}$$

or

$$\rho_{\text{now}} \sim m \left(\frac{M_0}{m} \right)^2 \left(\frac{mc}{\hbar} \right) R_{\text{now}}^{-2}$$

where $m \sim 10^{-27}$ gm is the electron mass; $mc/\hbar \sim 10^{-10} \text{ cm}$; $M_0/m \sim 10^{23}$; and $(4\pi/3)R_{\text{now}}^3 \sim 4 \times 10^{85} \text{ cm}^3$, or $R_{\text{now}}^2 \sim 10^{57} \text{ cm}^2$. The result is :

$$\rho_{\text{now}} \sim 10^{-28} \text{ gm/cm}^3 \quad (6.1)$$

which is within one order of magnitude of the desired astrophysical result to account for the apparent acceleration of the Universe. However, if ξ had been chosen as $\sim .1$, rather than ~ 1 , the $\rho^{1/4}$ values at initial and final inflation times would not be significantly changed,

but ρ_{now} would become 10^{-29}gm/cm^3 which is precisely the order of magnitude to fit the acceleration data.

The above simple, order of magnitude estimates clearly suggest that, within this QED vacuum energy theory, inflation and dark energy have the same origin; and that the word “Remnant” of the title is appropriate. According to this picture, dark energy and the acceleration it causes, decrease as the Universe expands.

7 Summary and speculation

The above Sections have defined a simple, intuitive picture of inflation and its remnant dark energy based upon a new, symmetry breaking/bootstrap model of a QED “vacuum energy”. Use of the simplest lepton bubble fluctuations allowed a Gaussian functional operation to be performed; but there are surely small, perturbative corrections to our bootstrap equation for A_μ^{vac} . Quite apart from numerical approximations, the basic idea of such a QED vacuum field and source of residual energy, well defined and everywhere finite, represents a slight enlargement of standard QED, one which cannot be negated by any test at current particle energies. It is a new possibility; and even a cursory glance at Fig. 1 suggests a possible connection with inflation.

Of astrophysical interest are the speculations which this model suggests : that there could be massive, charged tachyons T and \bar{T} , with a pole and perhaps cuts sitting unexpectedly, far out on the left hand side of a Lehmann representation for the photon propagator; and that such $T - \bar{T}$ pairs, tightly bound together, could also be present in the quantum vacuum, with Schwinger mechanism liberation in situations of astrophysical catastrophe, such as a supernova explosion. Such speculations by the authors can be found in a recent arXiv listing [2], where, in particular, it is noted that such charged tachyons of high energy – and therefore of velocity just slightly larger than c – are perfect candidates for dark matter; and they could easily be trapped in known, galactic magnetic fields on the order of 10^{-6} Gauss, that are coherent over huge distances on the order of 10^4 parsecs.

The A_μ^{vac} described in the present paper does not arise from a well defined set of charges and currents over which one has control; this is an acausal field, a property of the quantum nature of every point in space. On the basis of a simple, Born approximation calculation, it is interesting to note that such a field cannot scatter an ordinary, electrically charged particle unless the energy of that particle is on the order of M ; and since bubbles of the lepton family conspire to produce an M on the order of 10^8 GeV, one need not search for such scattering processes at the LHC. In contrast, such a field could, in Born approximation, scatter a charged tachyon of mass M through an angle θ satisfying $\cos \theta = 1/2$; and therefore, if the Big Bang contained a burst of such tachyons and anti-tachyons along any arbitrary axis, they could, by now, be distributed everywhere in the Universe.

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